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A heuristic method for the inventory control of short life-cycle products

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Abstract

In many manufacturing sectors, the product life cycle is becoming increasingly shorter. The direct application of standard forecasting and inventory control techniques in short life-cycle products (SLCPs) is not effective because their demand is transitory, non-stationary, and highly variable. In this paper, we develop an easy-to-implement heuristic method for the inventory control of SLCPs during their sales season. The heuristic method comprises a single-shipment submodel at the beginning of the season and a submodel that considers multiple shipments through the season. Both models work under a Vendor Managed Inventory (VMI) environment within a one-warehouse *N*-retailer supply chain, and attempt to minimize the total relevant cost of the system, which consists of the cost of returns, shortage costs, and fixed shipping costs. The latter costs have been barely considered in the literature for SLCPs. In the multiple-shipment submodel, we determine the shipment size by adapting the well-known Economic Order Quantity (EOQ) model. The safety stock is determined by means of the critical fractile of the newsvendor problem. Based on real sales data of a textbook publishing company, we compare the behavior of the heuristic method with the current shipping strategy of the firm. In all the test cases, the total relevant cost produced by the heuristic method was less than that of the current policy control of the company.

Keywords: Short life-cycle products, Inventory management, Newsvendor problem, Vendor managed inventory, Heuristics.

INGENIERÍA INDUSTRIAL

Un método heurístico para el control de inventarios de productos de corto ciclo de vida

Resumen

En diversos sectores manufactureros, el ciclo de vida de los productos es cada vez más corto. La aplicación directa de técnicas convencionales de pronóstico y control de inventarios a productos de corto ciclo de vida (SLCPs) no es efectiva porque su demanda es transitoria, no estacionaria y altamente variable. En este artículo, desarrollamos un método heurístico de fácil implementación para el control de inventarios de SLCPs durante su temporada de ventas. El método heurístico comprende un submodelo de un solo despacho al comienzo de la temporada de ventas y un submodelo que considera múltiples despachos a lo largo de dicha temporada. Ambos modelos trabajan bajo un ambiente de inventario manejado por el proveedor (VMI) dentro de una cadena de abastecimiento con una bodega y *N* detallistas, donde se busca minimizar el costo total relevante del sistema, compuesto por los costos de devoluciones, los costos de faltantes y los costos fijos de despacho. Estos últimos costos han sido escasamente considerados en la literatura para productos de corto ciclo de vida. En el modelo de múltiples entregas, se determina el tamaño de envío mediante una adaptación del conocido modelo de la cantidad económica de pedido (EOQ). El inventario de seguridad es determinado mediante la razón crítica del modelo del vendedor de periódicos. Con base en datos reales de ventas de una empresa productora de textos escolares, se compara el desempeño del método heurístico con la estrategia actual de despachos de la firma. En todos los casos de prueba, el costo total relevante del método heurístico fue menor que el de la actual política de control de la compañía.

Palabras clave: Productos de corto ciclo de vida, Gestión de inventarios, Problema del vendedor de periódicos, Inventario manejado por el proveedor, Heurísticos.

1. Introduction

1.1 Background

More demanding customer needs and requirements, increasing competitive pressures, and accelerated technological changes currently urge companies to adopt innovation as a key factor to ensure their sustainability. Commonly, four or five product cycle stages are considered: introduction, growth, maturity, saturation, and decline. For many products, the maturity and saturation stages last for a long time. For other products, their full life cycle might last for less than a year or even for a shorter period of time. Kurawarwala & Matsuo (1996), Kapuscinski et al. (2004), and Nair & Closs (2006) state that, at present, it is common to find products whose demand only occurs during a short period of time, after which the products usually become obsolete even if their intrinsic characteristics or quality features do not change. Such products are known as short life-cycle products (SLCPs).

Given the long manufacturing and distribution lead times that SLCPs might have and their high transportation and fixed production costs, production orders and purchase decisions are usually made prior to the sales season when information may be very limited. When shortages occur during the sales season, it is possible that no inventory replenishments can be done. Therefore, the retailer and the manufacturer usually hold large inventories at the beginning of the season. On the other hand, holding large inventories may result in excessive costs of obsolescence because SLCPs cannot be regularly sold in future seasons. In some cases, when SLCPs are shipped to other locations where they can still be sold, companies must pay significant relocation costs. In some manufacturing sectors, the combined shortage and obsolescence costs exceed manufacturing costs (Fisher & Raman, 1999; Nair & Closs, 2006), and in many cases the products become obsolete only after several time periods (Silver et al., 1998). In these cases, it is possible to place more than one order during the sales season.

Companies that used to produce products with long life cycles, have nowadays to manage very short life cycles and a large variety of products. Lee (2002) says that because SLCPs have transient and highly erratic demand, their design, manufacturing, physical distribution, and inventory management are difficult to execute. Some of the sectors that are most affected by the short life cycle of their products are: computer manufacturing, cell phones, garments, and the publishing industry.

According to Benjaafar et al. (2005), in a decentralized system with N identical and independent locations, the amount of inventory varies linearly with N, whereas, in a centralized system, the amount of inventory is approximately proportional to $\sqrt{N}$. Consequently, in a SLCP system, centralized decisions and collaborative strategies, inventory consolidation, and product postponement can generate important benefits by lessening the bullwhip effect and reducing shortages, excess inventory, safety stock, and the risk of obsolescence. Vendor Managed Inventory (VMI) is a very good alternative to establish such collaborative strategies because it attempts to solve coordination problems by giving the supplier the responsibility to manage the replenishment process and decide when and how much to ship to the customer (Kaipia et al., 2002). VMI can reduce inventory levels and simultaneously improve customer service. It is a way to improve total channel profit, particularly when there are disagreements in the inventory systems of the retailer and the supplier, which is usually the case in the publishing industry (Yao et al., 2007; Dong & Xu, 2002).

1.2 Relevant literature review and problem definition

As stated by Kurawarwala & Matsuo (1996), the literature on demand forecasting and inventory management does not adequately address topics about SLCPs. Nair & Closs (2006) state that traditional forecasting techniques may not be useful because most of them require at least one year of data. Even if these data were available, they would not generate satisfactory forecasts because some of the forecast technique assumptions do not hold. In addition, forecasting becomes more complex when price adjustments and advertising campaigns are implemented during the sales season.

Fisher et al. (2000) state that sales during the first part of the season are a good predictor of the total sales of a new product. Empirical results suggest that forecasts based on early sales are much more accurate than subjective predictions (Fisher & Raman, 1999). Rodríguez (2007) proposes a forecasting method based on the linear regression of cumulative demand through the sales season, using data from the demands of products with similar life cycle to estimate the regression parameters.

When the length of the season is much shorter than the replenishment time or when replenishment costs are high, a widely-studied approach has been to make a single shipment at the beginning of the sales season for a quantity that optimally balances the costs of understock and overstock. This approach is the well-known newsvendor or newsboy problem, which has had numerous extensions (e.g., Chung & Flynn, 2001; Matsuyama (2006); Chung et al., 2008).

Other researchers have considered multi-shipment scenarios. Kurawarwala & Matsuo (1996) involve a stochastic inventory model with finite horizon to determine the optimal replenishments along the life cycle. This model disregards fixed transportation and order processing costs. Zhu & Thonemann (2004) propose a heuristic to determine near-optimal inventory policies for SLCPs. Their model includes an ordering cost per unit and the holding and shortage costs, but does not consider fixed ordering costs either. Chen et al. (2007) formulate a nonlinear mixed-integer programming model to determine the optimal number of replenishments over a finite planning horizon. They assume a deterministic curve of demand and do not consider shortage costs.

Few real SLCP cases have been reported in the literature. For instance, Kapuscinski et al. (2004) present a forecasting technique and a system for controlling the inventory of components. The system has been implemented at Dell Computer Inc. and has helped to save at least 43 million dollars of inventory cost of one component employed in computer manufacturing. As Wagner (2002) states, analysts have made many efforts to develop effective inventory control methods that are easy to apply. However, customers continue to deal with empty shelves and excessive shortages appear to be unavoidable. Consequently, and given the complexity of a SLCP inventory control system, we believe that it is necessary to develop heuristic methods that can be easily applied to real cases and that yield satisfactory results. We have found that these methods outperform empirical inventory control strategies typically found in practice. Accordingly, in the present work, we develop an easy-to-implement heuristic method to control the retailer inventory of SLCPs under a VMI environment within a supply chain with a manufacturing plant, one warehouse, and N retailers. In contrast to the majority of previous works, our model includes fixed ordering costs in the decision process and is applied in a real case. The heuristic has been divided into two parts. First, based on the newsvendor model, we consider a single shipment at the beginning of the sales season. Second, we allow the system to have two or more shipments during the sales season, based on the forecast of the remnant demand. The heuristic then selects the policy that most likely produce the least total cost.

We focus on a real case of distributing textbooks from one warehouse to 34 different retailers. Currently, the organization does not have enough capacity to produce during the sales season and therefore, it must produce ahead of time. Our results confirm that the developed heuristic significantly improves current practices. The heuristic provides the shipment plan through the season and minimizes the amount shipped, the shortages and excess inventory, and the total relevant cost.

The rest of this paper is organized as follows. Section 2 describes the inventory control heuristic. In Section 3, we evaluate the behavior of the heuristic based on real data from a book publishing company, and discuss these results in Section 4. Lastly, in Section 5 we present some conclusions.

2. Development of the heuristic method

2.1 Model assumptions

The development of the heuristic method is based on the following assumptions:

• We consider a supply chain with one warehouse, and N retailers that share demand information and make centralized decisions, so facilitating the application of VMI concepts.

• Product demand is transient, stochastic, non-stationary, and non-correlated with the level of inventory at each retailer.

• The product cost, sale price, and replenishment costs are known and stable along the sales season.

• Any unsatisfied demand is considered as a lost sale.

• During the sales season, there are no product shipments among retailers.

• After the sales season, each unsold unit has a salvage value that is equal to a fraction of the unit sale price to the retailer. The salvage value includes the inventory holding cost during the season at the point of sale. This cost is assumed to be proportional to the number of units kept in inventory but not to the time that the products are maintained in stock. It is based on the assumption that sales season lasts for a short time.

2.2 The single-shipment submodel

We model the total demand during the sales season as a triangular probability distribution because it is a representation easily compatible with aggregate forecasting systems for short life cycle products. It also constitutes a simple representation of an unknown distribution that lies within certain bounds (Nair & Closs, 2006). It is therefore necessary, from pre-season information, to estimate the minimum expected or pessimistic demand a (the lower bound), the optimistic demand c (the upper bound), and the most likely demand b (the mode).

By using the known critical fractile of the newsvendor model (see, for instance, Chopra & Meindl, 2004) and the inverse cumulative distribution function of the demand $\Psi^{-1}(\cdot)$, we determine the optimal size of the shipment Q^* by means of Eqs. (1) and (2).

In Eq. (1), the unit cost of understock C_F is equal to the difference between the product cost v and the unit salvage value s. Since Eq. (1) is valid for any continuous nonnegative random variable, we can apply it to the inverse cumulative function of the triangular distribution, and obtain the optimal shipment size Q^* that maximizes the total expected profit (see Appendix A).

Given that we are looking for the best solution for the entire supply chain under a VMI environment, the parameters of the model must be defined from a global perspective, in order to avoid 'double marginalization'. This expression means that each of the supply chain parties perceives a different portion of the supply chain profit margin and thus that party makes its own decisions, reducing the total profit of the supply chain as a whole. To avoid double marginalization, therefore, one should consider p as the unit sale price to the final customer, v as the total unit variable cost paid by

$$Q^* = \Psi^{-1}\left(\frac{C_F}{C_F + C_E}\right) = \Psi^{-1}\left(\frac{p-v}{p-s}\right) \tag{1}$$

$$Q^* = \begin{cases} a + \sqrt{\left(\frac{p-v}{p-s}\right)(c-a)(b-a)} & , \quad 0 \le \left(\frac{p-v}{p-s}\right) < \left(\frac{b-a}{c-a}\right) \\ c - \sqrt{\left(1-\left(\frac{p-v}{p-s}\right)\right)(c-a)(c-b)} & , \quad \left(\frac{b-a}{c-a}\right) \le \left(\frac{p-v}{p-s}\right) \le 1 \end{cases} \tag{2}$$

$$\mathrm{E}(U \mid Q) = \begin{cases} Q(p-v) - \dfrac{(p-s)(Q-a)^3}{3(b-a)(c-a)} - A_0 & , \quad Q < b \\ \dfrac{(a+b+c)(p-s)}{3} - \dfrac{(p-s)(c-Q)^3}{3(c-b)(c-a)} + Q(s-v) - A_0 & , \quad Q \geq b \end{cases} \qquad (3)$$

the supply chain to place the product in the point of sale (including the variable transportation cost), and s as the net unit salvage value of the product after discounting the inventory holding cost during the season as well as the return and any product recovery costs. Q^* will then be the optimal shipment size for the entire supply chain.

Although the newsvendor model attempts to optimize order size, it does not explicitly determine whether it is convenient to actually ship the products. To overcome this shortcoming, before shipping the products, it is reasonable to verify whether the net profit remains positive after considering the fixed replenishment cost A_0, given in \$ / shipment. To be precise, the system will only ship the products if, for any shipment size Q, the expected value of the profit is greater than zero, that is:

$$\mathrm{E}(U \mid Q) > 0 \qquad (4)$$

We calculate this expected profit by using Eq. (3) (see Appendix B).

2.3 The multiple-shipment submodel

We now develop a submodel that operates under a VMI approach where the supplier knows the stock level at each retailer during the sales season. At the beginning of the sales season, we have a forecast of the minimum expected demand a by point of sale. As in the single-shipment submodel, it is necessary to check whether the expected profit of the first shipment compensates the fixed shipping cost. If the first shipment size is equal to the expected minimum demand a, then the condition that must be satisfied is:

$$a(p-v) \geq A_0 \qquad (5)$$

If this constraint does not hold, then the shipment size should not be equal to a ; in that case, we should apply the single-shipment submodel described in the previous section.

After making the first shipment, we should determine at each time period whether it is necessary to replenish. Under the multiple-shipment scenario, it is not easy to determine the expected profit of a specific shipment because, except for the last shipment of the sales season, any overstock at the end of a period cannot be considered as a product return. Consequently, instead of determining the economic feasibility of making each shipment, its size is calculated by using the Economic Order Quantity (EOQ) model. At each time period, the shipment size Q is defined by the interval:

$$\mathrm{Max}(Q_A, Q_B) \leq Q \leq Q_C \qquad (6)$$

where Q is the amount (in units) to ship if shipping is feasible, Q_A is the forecast of the amount (in units) required to satisfy the expected demand of the period plus the corresponding safety stock, Q_B is the economic order quantity (in units), and Q_C is the forecast of the amount (in units) required to satisfy the remaining expected demand of the sales season plus the corresponding safety stock.

We will consider the possibility for replenishment whenever $Q_A > 0$. We calculate Q_A based on the equation:

$$Q_A = \hat{x}_{j+R+L} + k\hat{\sigma}_{j+R+L} - I_j \qquad (7)$$

where I_j is the inventory level (in units) at the retailer at the end of period j, k is the safety inventory factor for the period, L is the replenishment lead time (in time units), R is the review interval (in units), $R \equiv 1$, $\hat{x}_{j+R+L}$ is the forecast demand (in units) from period $j+1$ to period $j+R+L$, and $\hat{\sigma}_{j+R+J}$ is the estimation of the standard deviation (in units) of the forecast error demand from period $j+1$ to period $j+R+L$, to calculate the corresponding safety stock.

As stated by Kapuscinski et al. (2004), the safety factor k can be determined by using the critical fractile of the newsvendor model:

$$k = \Phi^{-1}\left(\frac{C_F}{C_F + C_E{}'}\right) \tag{8}$$

where $\Phi^{-1}(\bullet)$ is the inverse cumulative standard normal distribution and $C_E{}'$ is the holding cost of an unsold unit between consecutive periods, given in \$ / (unit · period). Since any unsold unit in a period may be sold in the next period, we assume that

$$NC_E{}' = C_E \tag{9}$$

where N is the total number of periods into which the sales season has been divided. Since $NC_E{}'$ stands for the holding cost of a unit for the rest of the sales season, this expression is equivalent to the holding cost that the standard EOQ model contains. So, from Eq. (9) and the known EOQ expression, we obtain:

$$Q_B = \sqrt{\frac{2\hat{X}A_0}{C_E}} \tag{10}$$

where $\hat{X}$ is the forecast (in units) of the total demand of the season.

In addition, it is necessary to verify that Q_B plus the stock at the point of sale do not exceed the estimated amount to satisfy the demand of the remainder of the season and the corresponding safety stock. To accomplish this, we calculate the following quantity:

$$Q_C = \hat{x}_N + k_N\hat{\sigma}_N - I_j \tag{11}$$

where k_N is the safety inventory factor for the remainder of the season, determined by means of Eq.(8) by using C_E instead of $C_E{}'$, $\hat{x}_N$ is the forecast demand (in units) over the rest of the season, and $\hat{\sigma}_N$ is the standard deviation (in units) of the forecast errors of the remaining demand over the rest of the season, used to calculate the corresponding safety stock.

It is important to note that the economic order quantity Q_B may be slightly less than the amount necessary to satisfy the remnant demand of the season, Q_c. For instance, suppose that the required amount to satisfy next week's demand forecast plus the safety stock is equal to 20 units, that the EOQ is 91 units, and that the expected total demand including safety stock for the rest of the season is 98 units. If we decided to ship an EOQ of 91 units, then it would be likely that a possible upcoming shipment to satisfy the remaining demand would not be profitable.

However, since we know that the EOQ model is not very sensitive to lot size, we may define a range, i.e., from 90 % to 110 % of the optimum Q_B, to adjust the shipment size in order to reduce the risk of shortages in the last periods of the season. Specifically, in our heuristic, whenever $1.1\,Q_B > Q_C$, we set the shipment size equal to Q_C. If we ship this quantity, we can assume that it will not be necessary to make an additional shipment, unless an unexpected increase in demand occurs by the end of the season. Therefore, the expected understock and overstock of the shipment will then be the expected understock and overstock of the rest of the season. Accordingly, we can compare the expected profit of the shipment with the expected profit when nothing is shipped.

Similarly, the expected profit of the last shipment of the season should be calculated to determine whether such a shipment is profitable. We assume a normal distribution of the forecast errors of the demand through the end of the season. So, to calculate the expected profit U as a function of the necessary inventory T to fulfill the demand of the remainder of the season, we used Eq. (12) (see Appendix C).

$$E(U \mid T) = \left[\hat{x}_N - \hat{\sigma}_N\, G_z\left(\frac{T-\hat{x}_N}{\hat{\sigma}_N}\right)\right] p + \hat{\sigma}_N\, G_z\left(\frac{\hat{x}_N - T}{\hat{\sigma}_N}\right) s - vT - A_0 \qquad (12)$$

where $G_Z(\bullet)$ is the unit normal loss function, and:

$$T = \begin{cases} I_j & \text{when the shipment is not sent} \\ I_j + Q_j & \text{when the shipment is sent,} \end{cases} \qquad (13)$$

where Q_j is the amount (in units) to ship in period j, if shipping is fleasible.

To develop the heuristic for the multiple-shipment case, we need to define some additional variables: Fal is the variable to accumulate shortages (in units), I_0 is the initial inventory (in units) at the point of sale, j is the period counter, M is the shipment counter, Q_1 is the size (in units) of the first shipment, X is the variable to accumulate demand (in units), and x_j is the observed demand (in units) in period j.

Figure 1 shows the complete heuristic method. It is important to note that, according to our experiments, we have found that by defining the first shipment Q_1 equal to:

$$Q_0 = a + \sqrt{\frac{2A_0Q^*}{C_E}} \qquad (14)$$

we obtain a total cost lower than the cost corresponding to $Q_1 = a$. The rationale for this expression is the following. Since a is the minimum expected demand, it should be the minimum feasible initial shipment. However, this value should be adjusted by considering the optimal shipment Q^* and the associated fixed cost A_0, incorporating the triangular probability distribution of demand and the relationship between shortages and overstock. As shown in Eq. (14), we do this by adding a quantity derived from an adaptation of the EOQ model.

Another interesting result that we found is that, for all the cases we tested, the ratio Q_0 / Q^* is an indicator of whether the single-shipment submodel or the multiple-shipment submodel will produce the least total costs. This has been implemented in the heuristic by defining a threshold P that can be specified based on empirical results. In any particular case, however, the heuristic can be run for both submodels with preliminary data and then the user can estimate the critical value P that produces the best performance of the heuristic.

It is well known that the EOQ model is not highly sensitive to variations in the lot size. Therefore, we can establish the following empirical rule. Whenever $Q_0(1 + \alpha)$ is greater or equal than Q^*, then it is better to send a single shipment of optimal size Q^*. Therefore, the critical value P can be calculated as

$$P = Q_0 / Q^* = Q_0 / [Q_0 / (1 - \alpha)] = 1 / (1 - \alpha) \qquad (15)$$

3. Application of the heuristic method

3.1 Data

The current policy of the company is to make a single shipment at the beginning of the season, based on a simple empirical rule, i.e., the shipment amount is 125 % of the total expected demand.

To compare the behavior of the heuristic method with the inventory policies of the company, we used the data shown in Table 1. These data correspond to the sales of a SLCP in 34 retailers during the 2006 season, which lasted for 12 weeks approximately. Figure 2 illustrates the behavior of the sales at six randomly selected retailers. These data are representative because they reflect different life cycle patterns of products with different sales volume over the season. We used sales instead of demand data because the latter are extremely difficult to obtain from self-service retailers.

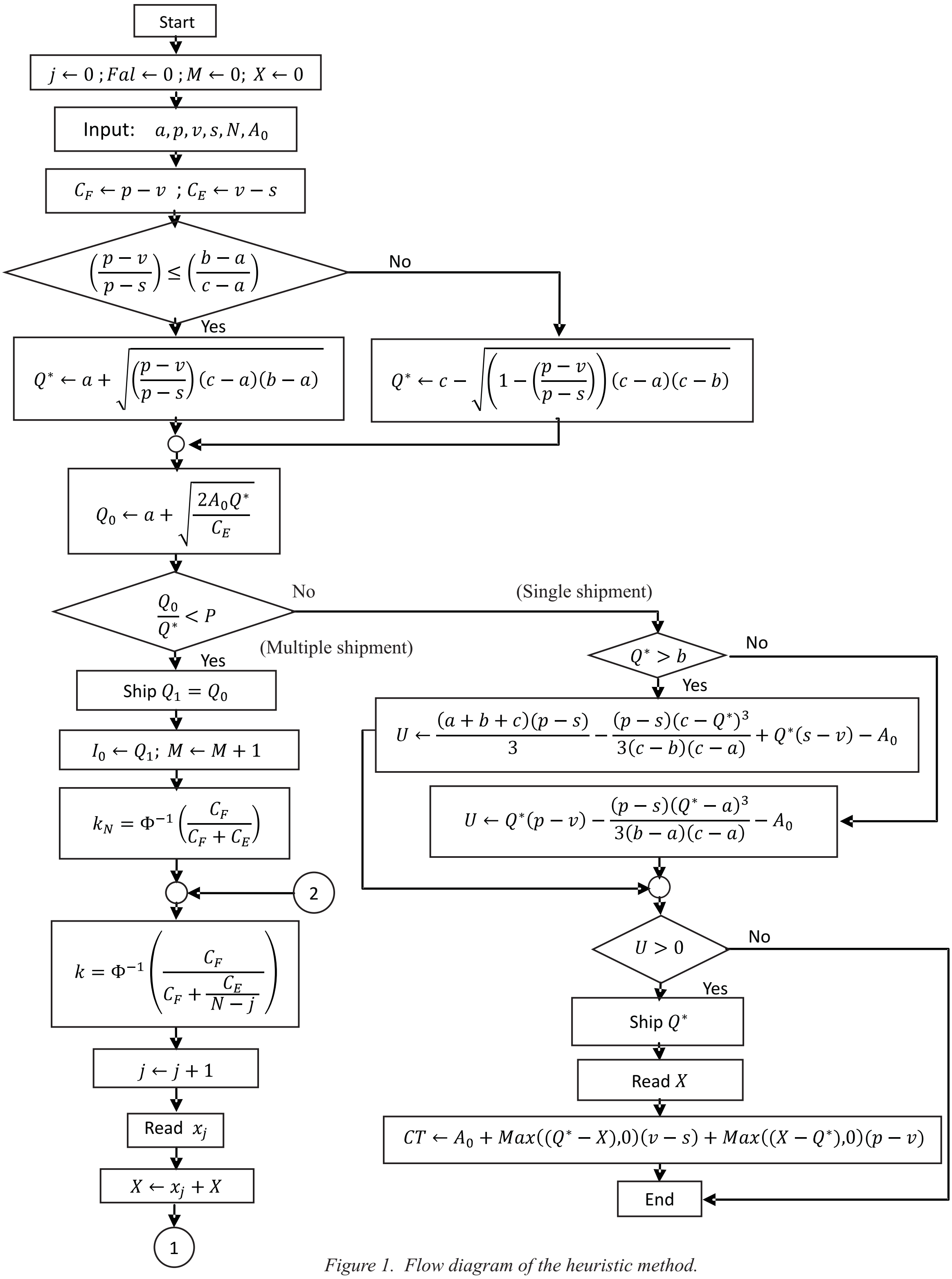

Figure 1. Flow diagram of the heuristic method.

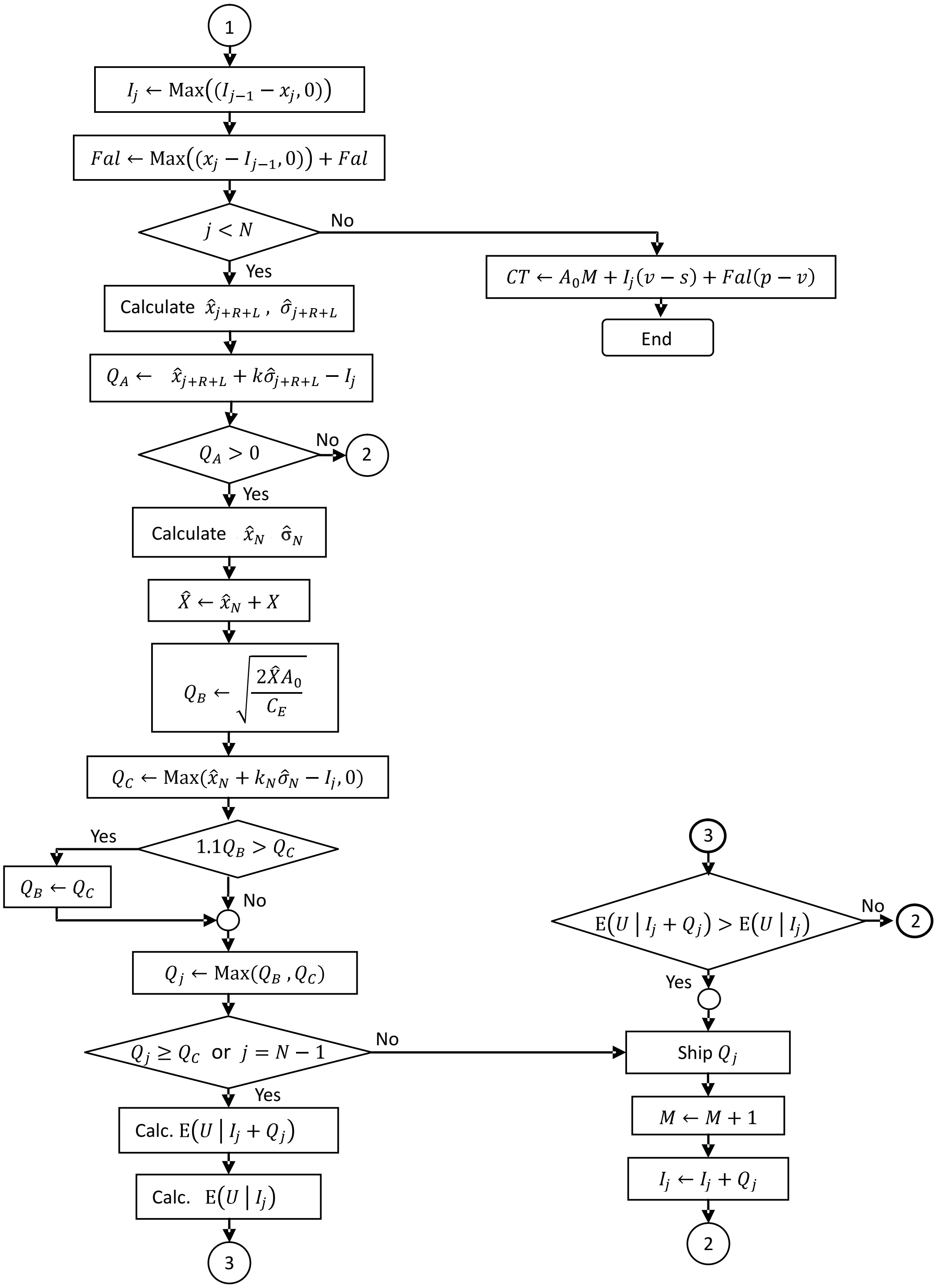

Figure 1. (continued)

Table 1. Weekly sales(in units) in 34 retailers during the 2006 season.

Retailer	WEEK 1	2	3	4	5	6	7	8	9	10	11	12	TOTAL
1	3	21	51	70	76	28	33	9	7	4	0	0	**302**
2	3	15	54	61	67	37	21	13	1	1	1	2	**276**
3	1	24	45	63	44	33	23	14	3	3	2	1	**256**
4	1	22	52	54	61	17	23	12	2	7	3	0	**254**
5	4	20	37	59	37	23	29	18	13	8	4	1	**253**
6	0	26	66	43	46	23	35	8	3	0	0	0	**250**
7	4	21	27	73	59	24	18	13	4	2	2	2	**249**
8	2	28	41	72	59	24	3	5	2	1	0	1	**238**
9	7	45	18	19	68	31	14	15	3	8	3	0	**231**
10	7	52	62	47	31	22	5	3	3	0	0	0	**232**
11	1	19	33	38	40	29	34	22	6	1	2	1	**226**
12	3	22	40	55	42	20	18	11	6	5	2	0	**224**
13	2	24	38	71	52	15	5	11	1	1	2	0	**222**
14	2	15	43	59	59	12	16	7	5	1	0	2	**221**
15	2	27	34	38	51	26	21	3	6	2	5	3	**218**
16	2	37	29	27	44	25	29	6	11	7	1	1	**219**
17	0	10	17	31	19	4	28	64	35	8	0	2	**218**
18	5	30	63	77	23	3	1	6	5	0	1	0	**214**
19	1	18	44	53	57	7	13	3	1	4	2	1	**204**
20	3	27	38	50	38	19	16	5	3	0	0	2	**201**
21	7	22	35	42	53	18	14	6	4	0	0	0	**201**
22	0	23	37	60	39	17	17	3	2	0	0	1	**199**
23	5	25	47	69	10	7	18	9	2	1	2	1	**196**
24	1	18	38	41	47	20	15	5	4	2	0	0	**191**
25	1	17	29	50	41	18	16	8	3	3	2	1	**189**
26	0	0	0	0	64	62	25	21	11	4	2	0	**189**
27	3	30	15	11	45	31	23	16	5	7	3	0	**189**
28	2	19	41	51	37	13	12	6	3	4	1	0	**189**
29	3	20	27	46	37	14	15	10	3	2	4	0	**181**
30	2	24	48	42	23	11	9	4	3	2	1	1	**170**
31	0	6	12	13	23	5	26	49	22	4	1	0	**161**
32	0	16	26	27	41	18	15	8	8	2	1	0	**162**
33	3	10	23	11	33	13	26	16	9	6	7	1	**158**
34	0	3	6	18	10	10	11	49	40	7	0	0	**154**
TOTAL	**80**	**736**	**1,216**	**1,541**	**1,476**	**679**	**627**	**458**	**239**	**107**	**54**	**24**	

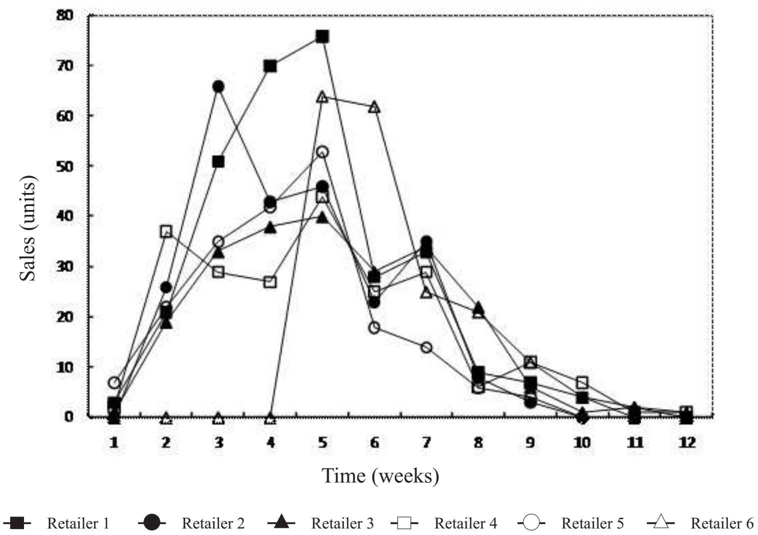

Figure 2. Weekly sales in six randomly selected retailers.
Note: the continuous lines facilitate the figure interpretation and do not represent continuous functions.

$$\hat{X}_m = \hat{\beta}_0 + \hat{\beta}_1 X_j + \hat{\beta}_2 X_{j-1}\ ; \text{ for } m = N, j+1\ ; \text{ where } j = 2,3,\dots,N-1 \tag{16}$$

3.2 Model parameters

A common practice of system and market analysts and demand planners is the direct parameter estimation, commonly based on information from other SLCPs because these products are almost always new and their demand information is very limited. In order to emulate the forecasting system of the company, we applied the STATFIT® software to obtain estimations of the pessimistic, optimistic, and most likely demands from the data in Table 1. Our results yielded α = 143 units, b = 189 units, and c = 311 units. In addition, we used the following parameters: p = 45 \$ / unit, s = 10 \$ / unit, A_0 = 20 \$ / shipment, L = 0, and R = 1 week. Implementing changes in these values is straightforward.

To forecast weekly demands and the total demand of the season at each retailer, as required by the multiple-shipment model, we applied a double-linear regression model of cumulative demands of 50 products with similar life cycle for the 2005 season. Eq. (16) was applied for that purpose.

where N is the number of periods of the season, X_j is the cumulative sales up to period j, $\hat{X}_m$ is the forecast of cumulative demand up to period m, $\hat{\beta}_0$ is the estimator of the independent term in the double-linear regression model, $\hat{\beta}_1$ is the estimator of the coefficient of cumulative demand up to period j in the double-linear regression model, $\hat{\beta}_2$ is the estimator of the coefficient of cumulative demand up to period j–1 in the double-linear regression model.

Eq. (16) indicates that the regression model must be applied for $m = N$ and for $m = j + 1$, because it is necessary to estimate the cumulative demand from period j to the end of the sales season and the demand of the next period.

This process is to be carried out from period j = 2 to period $j = N - 1$. In Table 2, we show the value of the estimators and the typical regression errors that we obtained. The typical regression errors were used in the multiple-shipment model as estimators of the typical forecast errors.

3.3 Computational results

We applied the heuristic method to the data of the 34 retailers shown in Table 1, and randomly selected six retailers to present our results.

Table 2. Parameters of the regression model based on sales of products with similar life cycle.

	Regression of sales for the whole season				Regression of sales for the next period			
	Typical regression error (units)	Parameters			Typical regression error (units)	Parameters		
Week		β_0	β_1	β_2		β_0	β_1	β_2
1	124.57	193.13	6.15		14.11	19.84	2.52	
2	83.45	61.98	−10.51	6.61	15.36	−3.95	0.28	2.34
3	80.33	62.81	2.79	1.59	10.47	11.26	0.61	1.26
4	52.62	–4.45	–5.80	5.68	16.73	–5.36	–2.21	2.88
5	20.95	8.13	–2.08	2.76	11.65	5.42	–1.10	1.98
6	8.52	–1.68	–0.89	1.82	5.74	–0.54	–0.46	1.44
7	4.77	–0.78	–0.69	1.70	3.64	–1.76	–0.51	1.52
8	2.33	1.40	–0.27	1.27	1.49	0.51	–0.19	1.19
9	1.55	0.64	–0.28	1.28	0.93	0.53	–0.12	1.11
10	1.09	–0.04	–0.26	1.26	0.93	–0.20	–0.23	1.23
11	0.46	0.19	–0.07	1.07	0.46	0.19	–0.07	1.07

Figure 3 illustrates the behavior of the inventory level at each selected retailer and Figure 4 shows the inventory levels when we forced the selected retailers to have a single shipment of optimal size during the season. From the two figures, it is clear that, when we applied the heuristic, the inventory level at the six chosen retailers, for most periods, was significantly less than that for the single-shipment approach. This fact has a positive effect on the total relevant cost, the level of returns, and the level of shortages, as will be noted below.

In addition, Figure 5 exhibits the distribution of shipments through the sales season for the six selected retailers after applying the heuristic. It is important to note that the shipments were distributed through the season in all cases. We did not observe a constant pattern of shipments for different retailers; this behavior suggests that the management of the model and its specific application to each retailer depend on the retailer, according to its particular demand through the season.

Table 3 shows the expected total relevant cost and the expected returns and shortages estimated for the current policy of the company, and contrasts them with those obtained from the application of the single-shipment submodel and of the heuristic to the shipment planning at the 34 retailers. The heuristic achieved a total relevant cost that is 52.7 % less than that of the current system and 36.1 % less than the cost obtained by applying the single-shipment submodel. These cost decreases are mainly caused by the decrease in returns yielded by the heuristic (63.3 % with respect to the current situation and 26.5 % regarding the single-shipment model).

It is important to note that the current policy reaches the least percentage of shortages with respect to the total amount shipped (0.51 %), but at the highest total relevant cost, total amount shipped, and returns. Under the single-shipment scenario, the shortage percentage is 2.6 %, whereas the percentage reached by the heuristic is 0.59 %, which is a satisfactory result.

4. Discussion

In Table 3, we illustrated the advantages of applying the designed heuristic for the inventory control of SLCPs. Now, we analyze the sensitivity of these results caused by different fluctuations in product price, salvage value, and fixed shipping costs.

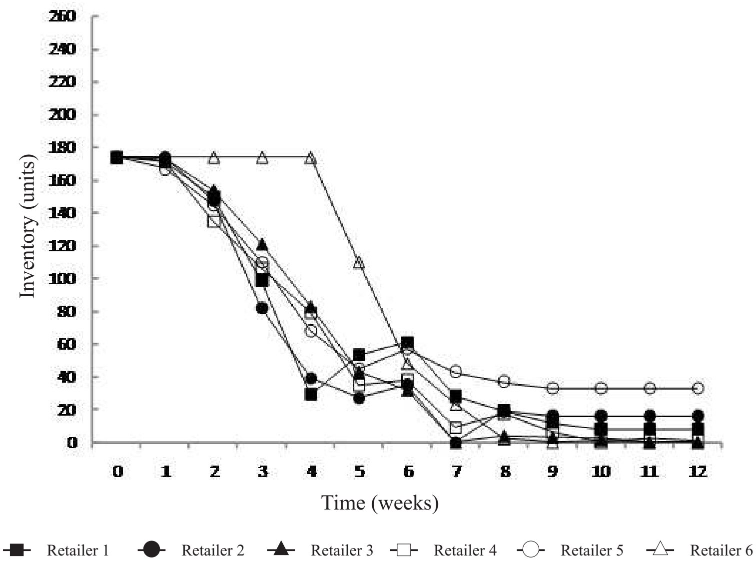

Figure 3. Inventory level at six retailers when applying the heuristic method.
Note: the continuous lines facilitate the figure interpretation and do not represent continuous functions.

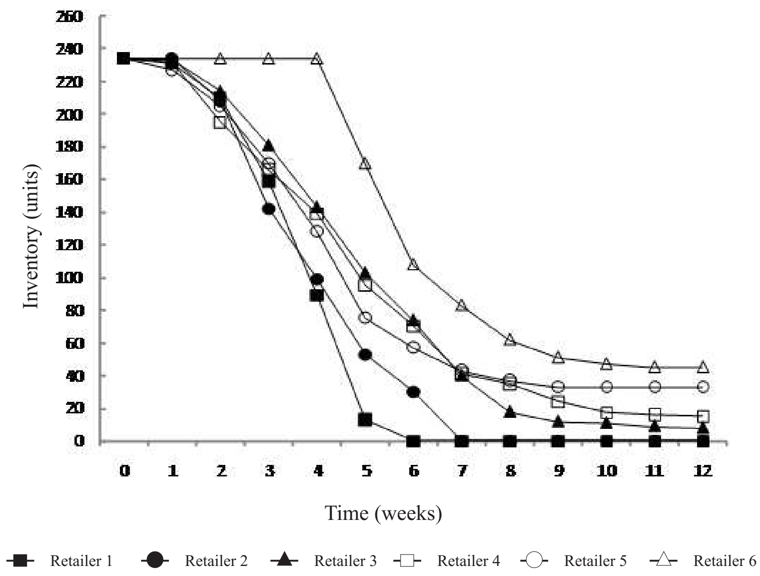

Figure 4. Inventory level at six retailers when applying the single-shipment submodel.
Note: the continuous lines facilitate the figure interpretation and do not represent continuous functions.

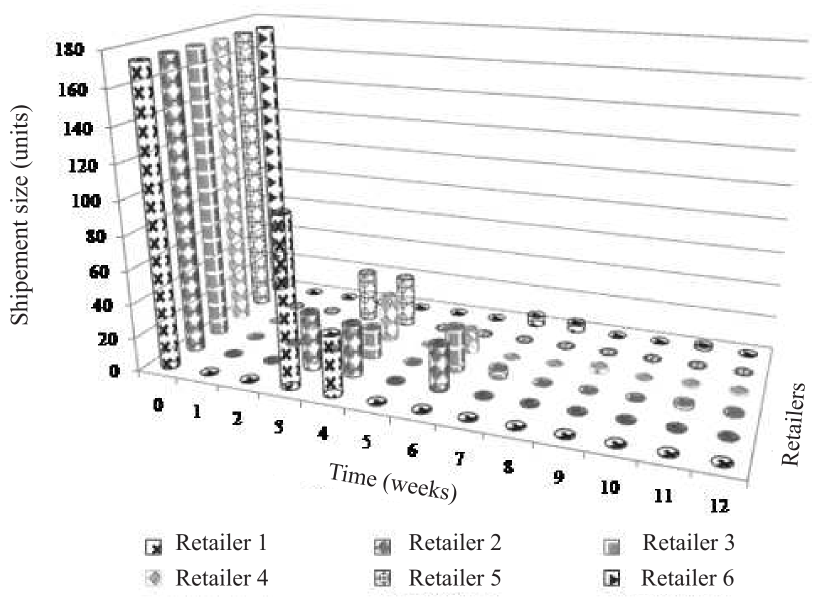

Figure 5. Size of the multiple shipments to six retailers through the sales season.

Table 3. Comparison between the heuristic method application, the current situation, and the single-shipment scenario.

Scenario	Total relevant cost ($)	Total amount shipped (units)	Returns (units)	Shortages (units)
Current situation	20,360	9,044	1,853	46
A single shipment of optimal size	15,080	7,956	925	206
Applying the heuristic method	9,630	7,841	680	46

The results are shown in Table 4, where the highlighted cells represent the least total cost for each combination of parameters. It is important to note that, in this case, by defining a threshold value $P = 0.87$ (representing an α value of 0.15), the heuristics determined whether the single-shipment submodel or the multiple-shipment submodel would produce the best results. This means that even if the company were to merely apply the newsvendor model to optimize a single shipment at the beginning of the season, the heuristic would produce at least equal or, for most cases, better results. Additionally, from Table 4 it is clear that the fixed shipping cost A_0 may have a significant impact on the best shipment approach and on the minimum total relevant cost.

Table 5 summarizes the response of the heuristic to extreme changes in the costs of overstock and understock. In this analysis, the sale price *p* has been kept constant. The changes in the costs of overstock and understock were obtained by varying the salvage value *s* or the unit variable cost *v*. The blank spaces in the table correspond to infeasible combinations of *p*, *v*, and *s*, since the latter would take on negative values.

As expected, when the costs of overstock or understock are negligible, the best is to apply the single-shipment approach. In this case, the critical value $P = 0.87$ is not a good predictor of the best submodel. Therefore, although it is not easy to find these cases in practice, if they occur, the single-shipment approach is highly recommended.

5. Conclusions

In this paper, we develop a heuristic method for the inventory control of products with a short life cycle in a one-warehouse *N*-retailer supply chain under a VMI environment. The heuristic combines single and multiple shipment approaches for the sales season. The developed models search for the best profit for the entire supply chain rather than for the particular interest of some of the links of the chain.

In the literature, we could not identify an inventory management model for SLCPs that considers fixed shipping costs under a multiple-shipment scenario. However, taking the fixed shipping cost into account is important as is shown in this work. The model exhibits a good behavior when the cost of shortages and returns are not negligible, which is the case in most real cases.

The heuristic provides the shipment plan through the season, automating the decisions of 'when to ship' and 'in what quantities'. Moreover, it allows a better inventory allocation through the supply chain, thus reducing total system inventory and the risk of obsolescence. By applying the heuristic in a textbook publishing company with 34 retailers, we could significantly reduce the total relevant cost, the amount shipped, the shortages and the excess inventory, which is precisely one of the most complex problems of the inventory control of SLCPs. Implementing the heuristic on electronic sheets is relatively simple, which can be very useful for the inventory management of many SLCPs.

Table 4. Response of total relevant cost ($) for the optimal single shipment and for the heuristic method to variations in price p, salvage value s, and fixed shipping cost A_0.

p ($/u)	s ($/u)	Q^* (u)	Scenario	Fixed shipping cost A_0 ($) 0	30	60	90	120	150
65	15	266	Single-shipment	11,335	12,355	13,375	14,395	15,415	16,435
			Heuristic	7,985	8,975	12,445	14,395	15,415	16,435
			Q_0 (units)	143	199	223	241	256	269
			Q_0/Q^*	0.538	0.748	0.838	0.906	0.962	1.011
	5	239	Single-shipment	23,355	24,375	25,395	26,415	27,435	28,455
			Heuristic	14,115	14,970	21,030	23,145	27,060	28,455
			Q_0 (units)	143	174	187	197	205	212
			Q_0/Q^*	0.598	0.728	0.782	0.824	0.858	0.887
45	15	253	Single-shipment	9,105	10,125	11,145	12,165	13,185	14,205
			Heuristic	5,965	8,165	11,145	12,165	13,185	14,205
			Q_0 (units)	143	198	221	238	253	266
			Q_0/Q^*	0.565	0.783	0.874	0.941	1.000	1.051
	5	223	Single-shipment	17,775	18,795	19,815	20,835	21,855	22,875
			Heuristic	11,060	13,510	18,175	20,835	21,855	22,875
			Q_0 (units)	143	173	185	195	203	210
			Q_0/Q^*	0.641	0.776	0.830	0.874	0.910	0.942
25	15	210	Single-shipment	4,745	5,765	6,785	7,805	8,825	9,845
			Heuristic	2,920	5,765	6,785	7,805	8,825	9,845
			Q_0 (units)	143	193	214	230	243	255
			Q_0/Q^*	0.681	0.919	1.019	1.095	1.157	1.214
	5	187	Single-shipment	7,115	8,135	9,155	10,175	11,195	12,215
			Heuristic	5,615	8,135	9,155	10,175	11,195	12,215
			Q_0 (units)	143	170	182	190	198	204
			Q_0/Q^*	0.765	0.909	0.973	1.016	1.059	1.091

Table 5. Response of total relevant cost ($) and Q_0/Q^* ratio for a single shipment and for multiple shipments to variations in the costs of overstock (C_E) and understock (C_F).

Cost of overstock C_E ($)	Scenario	Cost of understock C_F ($) 1	15	30	44
1	Single-shipment	1,629	3,241	3,660	3,843
	Multiple-shipment	2,384	3,527	3,632	3,814
	Q_0/Q^*	0.829	0.633	0.611	0.600
15	Single-shipment	2,707	14,915	20,150	
	Multiple-shipment	4,592	12,895	12,690	
	Q_0/Q^*	1.055	0.829	0.763	
30	Single-shipment	2,697	19,340		
	Multiple-shipment	6,826	17,245		
	Q_0/Q^*	1.094	0.897		
44	Single-shipment	2,703			
	Multiple-shipment	8,644			
	Q_0/Q^*	1.115			

6. Appendices

6.1 Appendix A

The probability density function of the triangular distribution ($a<b<c$) is:

$$f(x)=\begin{cases} f_1(x)=\dfrac{2(x-a)}{(c-a)(b-a)} & , \ a\le x<b \\[2ex] f_2(x)=\dfrac{2(c-x)}{(c-a)(c-b)} & , \ b\le x\le c \end{cases} \tag{A1}$$

Since

$$P(x\le X)=F(x)=\int_0^x f(x)dx \tag{A2}$$

the cumulative distribution function is given by:

$$F(x)=\begin{cases} \displaystyle\int_a^x f_1(x)dx & , \ a\le x<b \\[2ex] \displaystyle\int_a^b f_1(x)dx+\int_b^x f_2(x)dx & , \ b\le x\le c \end{cases} \tag{A3}$$

When solving the integrals in Eq. (A3), including the intervals where the density function is not defined, we obtain:

$$F(x)=\begin{cases} 0 & , \quad a<x \\[1ex] \dfrac{(x-a)^2}{(c-a)(b-a)} & , \quad a\le x<b \\[2ex] 1-\dfrac{(c-x)^2}{(c-a)(c-b)} & , \quad b\le x\le c \\[1ex] 1 & , \quad x>c \end{cases} \tag{A4}$$

This result may be used to get the value of x from a given cumulative probability R such that $R=P(X \fallingdotseq x)$. Finding x, we finally get

$$F^{-1}(R)=\begin{cases} a+\sqrt{R(c-a)(b-a)} & , \quad 0\le R<\dfrac{b-a}{c-a} \\[2ex] c-\sqrt{(1-R)(c-a)(c-b)} & , \quad \dfrac{b-a}{c-a}\le R\le 1 \end{cases} \tag{A5}$$

6.2 Appendix B

Consider the triangular probability density function described in Appendix A above. To determine the expected returns and the expected shortages, two cases must be taken into account:

6.2.1 Case $Q<b$

Since Q is less than the most likely value of demand, the expected returns are given by:

$$\mathrm{E}(Q-x)=\int_{-\infty}^{Q}(Q-x)f(x)dx \tag{B1}$$

By substituting the function on this interval, we get:

$$\mathrm{E}(Q-x)=\int_a^Q (Q-x)\frac{2(x-a)}{(c-a)(b-a)}dx \tag{B2}$$

which is equivalent to:

$$\mathrm{E}(Q-x)=\frac{(Q-a)^3}{3(c-a)(b-a)} \tag{B3}$$

To determine the expected value of shortages, we may consider:

$$\int_{-\infty}^{\infty}(x-Q)f(x)\,dx=\int_{-\infty}^{Q}(x-Q)f(x)\,dx+\int_{Q}^{\infty}(x-Q)f(x)\,dx \tag{B4}$$

Now,

$$\int_{Q}^{\infty}(x-Q)f(x)\,dx=\int_{-\infty}^{\infty}(x-Q)f(x)\,dx+\int_{-\infty}^{Q}(Q-x)f(x)\,dx \tag{B5}$$

For the triangular distribution

$$\int_{-\infty}^{\infty}(x-Q)f(x)\,dx=\frac{(a+b+c)}{3}-Q \tag{B6}$$

and from Eqs. (B1) and (B3), we get to the final result:

$$\mathrm{E}(x-Q)=\frac{(a+b+c)}{3}-Q+\frac{(Q-a)^3}{3(c-a)(b-a)} \tag{B7}$$

6.2.2 Case $Q\ \equiv b$

Here, we follow a similar procedure as in the previous case, that is

$$\mathrm{E}(x-Q)=\int_{Q}^{\infty}(x-Q)f(x)dx \tag{B8}$$

By substitution, we get:

$$\mathrm{E}(x-Q)=\int_{Q}^{c}(x-Q)\frac{2(c-x)}{(c-a)(c-b)}dx, \tag{B9}$$

That is, the expected shortages are:

$$\mathrm{E}(x-Q)=\frac{(c-Q)^3}{3(c-b)(c-a)} \tag{B10}$$

Now, following a similar procedure, the expected returns will then be:

$$\mathrm{E}(Q-x)=\frac{(c-Q)^3}{3(c-b)(c-a)}+Q-\frac{(a+b+c)}{3} \tag{B11}$$

The expected profit can be calculated as the sum of the revenues from the units we expect to sell, plus the revenue from the units returned at the end of the season, minus the cost of the purchased units, minus the fixed shipping cost. This is equivalent to:

$$\mathrm{E}(U)=[\mathrm{E}(x)-\mathrm{E}(x-Q)]p+\mathrm{E}(Q-x)s-vQ-A_0 \tag{B12}$$

By substituting the above expressions for E($x-Q$) and E ($Q-x$) in Eqs. (B7) and (B11), respectively, into Eq. (B12), the expected profit (given the shipment size Q) is given by the following equation, which corresponds to Eq. (4):

$$\mathrm{E}(U\mid Q)=\begin{cases} Q(p-v)-\dfrac{(p-s)(Q-a)^3}{3(b-a)(c-a)}-A_0 & ,\ Q<b \\ \dfrac{(a+b+c)(p-s)}{3}-\dfrac{(p-s)(c-Q)^3}{3(c-b)(c-a)}+Q(s-v)-A_0 & ,\ Q\ge b \end{cases} \tag{B13}$$

6.3 Appendix C

Given a random demand x with normal probability distribution, with standard deviation σ and expected value $\bar{x}$ for a given level of available inventory I , the expected number of shortages is given by (Thomopoulos, 1980):

$$E(x-I)=\int_{I}^{\infty}(x-I)f(x)=\sigma G_z(k) \tag{C1}$$

where f (•) is the normal probability density function, G_z (•) is the unit normal loss function, and k is the standard normal variable, such that

$$k=\frac{I-\bar{x}}{\sigma} \tag{C2}$$

Similarly, the expected number of returns is given by:

$$E(I-x)=\sigma G_z(-k) \tag{C3}$$

The expected profit, given an available inventory I can be calculated as follows:

$$E(U\mid I)=[E(x)-E(x-I)]p+E(I-x)s-vI-A_0 \tag{C4}$$

where *p, s, v,* and A_0 have been already defined in Section 2.3.

By substituting Eqs. (C1)-(C3) into Eq. (C4) be obtain Eq.(C5) which matches Eq.(12) with its corresponding variables:

$$\mathrm{E}(U \mid I) = \left[\bar{x} - \sigma\, \mathrm{G_z}\left(\frac{I-\bar{x}}{\sigma}\right)\right] p + \sigma \mathrm{G_z}\left(\frac{\bar{x}-I}{\sigma}\right) s - vI - A_0 \quad \text{(C5)}$$

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