

Optimal economic project selection under uncertainty: An illustration from an utility company

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Abstract

The problem of selection and management of projects is common in the planning process of private and public companies and is required to manage and allocate scarce resources usually between alternatives that differ in technical, operational, and financial aspects, in addition to the level of risk. This article presents an application of a portfolio optimization problem of projects under uncertainty and budget constraints. Firstly, it addresses the problem using as the objective function the maximization of the Expected Net Present Value (ENPV) of the portfolio of projects. Secondly, the objective function is the maximization of two indicators of risk associated with the portfolio ENPV: Maximizing the Net Present Value (NPV) of the portfolio for a certain level of certainty and maximizing the probability of obtaining a feasible portfolio in economic terms. The methodology was developed in the framework of a research project funded by an utility company. To illustrate the proposed methodology, a suitable example, which assumes that the company can partially allocate resources to several projects from a pool of independent investment alternatives was used. New developments in the area of stochastic optimization can help to improve the quality of decisions to allocate limited financial resources, through the use of performance indicators related directly to the company's strategy. The results show the optimal allocation of financial resources to each project taking into account the uncertainty associated with different input variables, which are modeled using empirical probability distributions defined a priori by the analyst.

Palabras Claves: Investment analysis, Capital budgeting, Project selection, Linear programming, Monte Carlo simulation.

INGENIERIA INDUSTRIAL

Selección óptima de proyectos económicos bajo incertidumbre: Ilustración de una compañía de servicios públicos

Resumen

El problema de la selección y ordenamiento de proyectos es común en los procesos de planeación de empresas privadas y públicas que tienen la obligación de administrar y asignar recursos usualmente escasos, entre alternativas que difieren en aspectos técnicos, operacionales, financieros, además del nivel de riesgo. Este artículo presenta una aplicación del problema de optimización de portafolios de proyectos en condiciones de incertidumbre y restricción presupuestal. En primer lugar, se aborda el problema utilizando como función objetivo la maximización del valor presente neto esperado (ENPV) del portafolio de proyectos y en segunda instancia se propone como función objetivo la maximización de dos indicadores de riesgo asociados al ENPV del portafolio: Maximizar el NPV del portafolio para un cierto nivel de confianza y maximizar la probabilidad de obtener un portafolio factible en términos económicos. La metodología se desarrolló en el marco de un proyecto de investigación aplicada financiado por una empresa de servicios públicos. Para ilustrar la metodología planteada se hace uso de un ejemplo adaptado, en el cual se supone que la compañía puede asignar recursos en forma parcial a varios proyectos entre un conjunto de alternativas de inversión independientes. Los nuevos desarrollos en el área de la optimización estocástica permiten mejorar la calidad de las decisiones de asignación de recursos financieros limitados, mediante el uso de indicadores de desempeño relacionados en forma directa con la estrategia de la empresa. Los resultados muestran la asignación óptima de recursos financieros a cada proyecto teniendo en cuenta la incertidumbre asociada a las diferentes variables de entrada, las cuales se modelan mediante distribuciones empíricas de probabilidad definidas a priori por el analista.

Keywords: Análisis de inversiones, Presupuesto de capital, Selección de proyectos, Programación lineal, Simulación de Monte Carlo.

1. Introduction

The allocation of financial resources to portfolios of projects is a common problem. Managers have to choose them wisely to fulfill their strategic objectives through the execution of these projects. Financial planners attempt to select an “optimal subset” of projects that increases the value of the company, while complying with the allocation and the specified budget constraints keeping a desired level of risk.

One of the most widely studied problems in Finance and Engineering Economics is the project selection and configuration of investment portfolios. This problem has been studied with traditional methodologies such as Net Present Value (NPV), Internal Rate of Return (IRR), Modified IRR and the Value of Future Cash Flows. All these techniques, if appropriately applied, lead to good decisions in terms of the projects to be selected. Also, some Operations Research models have been studied, using the maximization of profit or value as the objective function and subject to technical and budgetary constraints. Despite the effectiveness of all these methods, it is relevant to consider methods that address the uncertainties that are present in real life projects. The strategic importance of project selection processes is the critical need to implement projects that are geared towards the sustainability of the business in the long term, due to financial constraints and the uncertainty associated with the cash flows of each project, (Powers et al, 2002)

Traditional financial theory has developed portfolio selection techniques for financial assets, using the mean variance optimization as the dominant criterion (Markowitz (1952, 1959)). Modern portfolio theory is still supported by some of the principles proposed in Markowitz's model, such as the assumption that the returns from portfolios of financial assets are normally distributed (McVean, 2000). However, some empirical research projects have found that these returns do not behave in this fashion. This situation is especially important when considering portfolios that include investment in real projects, which are exposed to various sources of uncertainty, April et al. (2002).

Lorie & Savage (1955) presented pioneering work in project selection and ranking. They used Linear Programming to select investment alternatives subject to budgetary constraints. These authors also questioned the validity of the IRR measure versus the NPV as a selection criterion. This question has been extensively discussed in the engineering economics literature. Based on their work, several other authors have addressed the project selection problem using multiobjective linear optimization techniques (Ringuest & Graves (1990)), integer programming techniques (Bebed-Dov (1965)) or goal programming (Mukherjee & Bera (1995)), and Kalu (1999) proposed a multiobjective programming model that was applicable to the petroleum industry, with the special characteristic of considering the uncertainty associated to certain input variables of the model.

In a broader working environment, it is widely recognized that the problem of project selection must consider additional aspects of the economic and financial component. For this reason, some authors such as Nowak (2005) proposed the use of multicriteria techniques to link elements of qualitative and quantitative nature of the problem of selection.

Some applications of the problem of selecting portfolio of projects have been developed in industries with high levels of uncertainty, such as the oil industry. An example is the work of Sira (2006) who applies a scatter search algorithm to find a portfolio efficient frontier.

The development that evolutionary algorithms for stochastic optimization have experienced has made it possible to revise the investment scheduling to incorporate uncertainty in certain critical variables. Sometimes, the uncertainty cannot be modeled analytically, so other techniques are employed. It is very common that an investment project has external variables that are outside of the company's control, and internal variables that are related to technical and operational decisions.

Today, there are more powerful optimization algorithms, which provide a series of simulations

to produce high quality solutions of problems that cannot be solved analytically, (Pichittlamken & Nelson, 2001).

This paper addresses the process of financial resource allocation to real projects, using decision criteria that take into account and quantify the inherent risk present in these projects. To achieve this, issues such as uncertainty in the project's revenues, project margin and working capital requirements are considered. To illustrate the development of the proposed methodology, the information supplied by an utility company was used.

2. Methodology

There are two main sections in this paper. In the first one, the economic analysis of a set of investment projects is performed, considering the uncertainty in the input variables of each project. In this first section, Monte Carlo simulation is used. In the second section, stochastic optimization algorithms are applied to determine the project subset that maximize the ENPV of the project portfolio.

2.1 Construction of the project model

This stage refers to the construction of the computational model of the project. This model should represent the possible behavior that the decision criteria will have under different scenarios determined by the input variables.

According to Sefair & Medaglia. (2005), the NPV of a project could be presented as its discounted cash flow, affected by a random component ω , which can be estimated using simulation methods based on the uncertainty associated with the input variables in the model analysis, as follows:

$$NPV_i = \sum_{t=0}^{v_i} \frac{CF_{it}(\omega)}{(1+r)^t} \quad (1)$$

Where CF_{it} is the cash flow of the project i in the period t , r is the cost of capital and v_i is the project life i .

The construction of the evaluation model starts with the definition of the input variables. The different values that these input variables can take on will condition the results that the model will present. Input variables can be external when macroeconomic, market, and industry sector information is used; and internal when they refer to operational issues such as production capacities, operation times, inventory policies, and efficiency levels, to mention just a few. Input variables are necessary to define the costs and benefits of the alternatives that are under study.

Then, the relationships among variables must be established. As a result of these relationships, the cash flow of the project is defined. The cash flow is the cornerstone of the evaluation of the project. Once the cash flow is known, economic decision criteria can be applied, such as NPV, IRR,

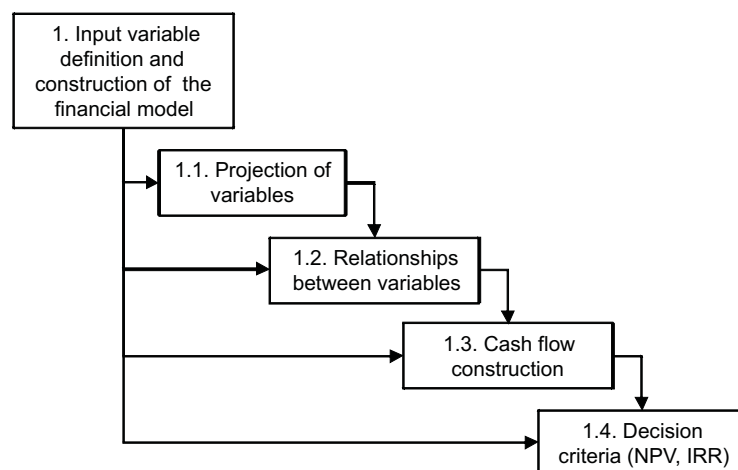


Figure 1. Project model construction process

modified IRR and discounted payback period, among others (Figure 1).

2.2 Simulation model of the project

The next step is to simulate the model. Using the computational model built, the impact of the input variables on the decision criteria is evaluated. There are many software tools available that enable simulation processes in spreadsheet-based models (@Risk, Crystal Ball, Insight.xla, SimTools.xla, EasyPlanex). All these tools provide a common work basis, since they allow the user to define uncertainty levels on the input variables, define correlations between variables, and assess sensitivity of these variables over the final results. They also present certain statistical analysis tools that can be applied on the decision criteria that will be used for the portfolio selection. In this paper, the software known as Crystal Ball version 11.1 in simulation analysis and OptQuest in Optimization – Simulation Analysis was used. (OptQuest is part of CrystalBall®, DecisionEngineering, 2004). The construction of the simulation model follows the steps outlined in Figure 2.

2.3 Definition of uncertainty in input variable

The uncertainty associated to each input variable in the model can be defined from probability

distributions known to the analyst, based on previous knowledge of the variable. This is a common practice in certain types of decisions. For example, to model the inter-arrival times in queueing system, the exponential distribution is assumed. However, it is possible to adjust the data to a certain probability distribution using statistical fit tools with tests such as Anderson-Darling, Kolmogorov-Smirnov, and Chi-square among others. The application of these tools would be possible if there were historical data available on the input variables.

It is common to find a certain degree of correlation among input variables in investment projects; therefore, this fact should be explicitly considered in the model. For example, the price-demand elasticity for a certain product affects the revenue of a project in the sense that small changes in price can cause large swings in demand. Correlations between input variables might be established by studying historical data or through assumptions made by the analyst. In this paper, the input variables considered are revenue increase caused by the project, the project's EBITDA margin, and the increase in working capital, measured as a percentage of the revenue of each project.

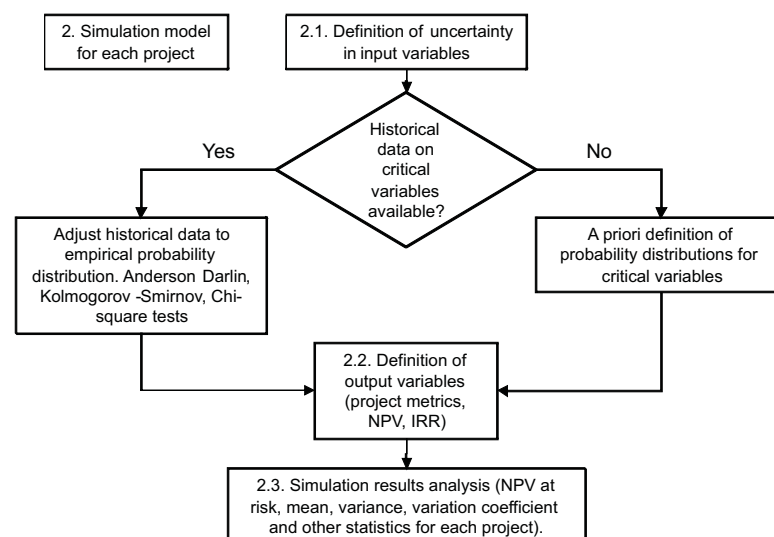


Figure 2. Characterization of critical variables

2.4 Definition of output variables

In this paper, the output variables will be the decision criteria applied to judge the economic qualities of the project. The ENPV of the portfolio will be calculated from the sum of the expected net present values of the various projects likely to be selected.

$$ENPV_{portfolio} = \sum_{i \in S} E(NPV_i) X_i \quad (2)$$

where X_i is a binary variable that takes the value of 1 if project $i \in S$ is selected; otherwise, it takes the value of 0. S is the set of projects to assess.

2.5 Simulation of the model

The last step in this first part of the paper is to obtain the simulation results. Since Monte Carlo simulation was used, relevant information is obtained for each output variable, such as a frequency histogram, mean and variance values, maximum and minimum values, kurtosis, and asymmetry and percentile values. This last element is very important for the analysis, since the objective function of the stochastic optimization model could be built to optimize one of these percentiles, i.e., the value at risk obtained for a certain confidence level, associated with the corresponding percentile.

2.6 Project portfolio optimization

In the search of optimal values for the budget allocation variables, the scatter search algorithm present in OptQuest® was used. Better & Glover (2006) propose a general framework for evolutionary algorithms when selecting financial alternatives, which is presented in the following expressions:

$$\begin{aligned} \text{Max, Min} &\rightarrow F(x) \\ \text{Constra int } s &\rightarrow Ax \leq b \\ \text{Re queriments} &\rightarrow g_l \leq G(x) \leq g_u \\ \text{Bounds} &\rightarrow l \leq x \leq u \end{aligned} \quad (3)$$

Where x is the budget allocation for a project (x can be a continuous or a discrete variable). In the model under consideration, x is the main decision variable. $F(x)$ represents the objective function, that is the value to be optimized, $G(x)$ represents the bounds associated to the objective function, and Ax are the technical and budgetary constraints that might be present in the model.

2.7 Construction of the objective function to be optimized

In this paper, three independent objective functions have been considered. The first one optimizes the ENPV of the portfolio and the second one has the NPV at risk as the objective. In the latter, the NPV of the portfolio is used by assuming a certain confidence level and an upper bound on the investment resources. The NPV at risk for an $\alpha\%$ confidence level could be defined as the value for which $\alpha\%$ of the possible NPV values are lower and $(1-\alpha)\%$ are higher. By using the NPV at risk as the decision criterion offers two perspectives for analysis: A project would be deemed feasible (with a $1-\alpha\%$ confidence level) if the value of the NPV at risk is greater than zero; in an analogous fashion, a project is considered attractive if the confidence level found for the zero NPV at risk is greater than or equal to the threshold confidence level expected by the decision maker. The third objective function incorporates the measurement of the risk associated with the probability of the non-feasibility of the projects that are part of the portfolio. In this case, the aim is to minimize the likelihood of obtaining negative values in the NPV of the portfolio.

Ye & Tiong (2000) present the definition of NPV at risk for an investment project in two different situations. In the first one, normality is assumed for the distribution of the decision criterion and the NPV at risk can be found by using the mean-variance method, with the difference between the mean and a multiple of the variance σ^2 of the NPV, shown as follows:

$$NPV \text{ at risk} = \text{mean NPV} - Z(\alpha) \sigma \quad (4)$$

Where $Z(\alpha)$ is the number of standard deviations that correspond to the confidence level. For example, for a confidence level of 95%, $Z(\alpha) =$

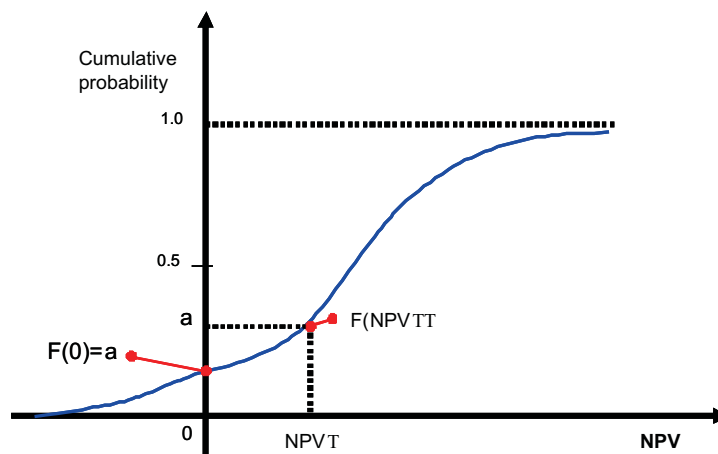


Figure 3. Estimation of the NPV at risk and the confidence level, from the cumulative probability function.

1.65. Assuming that the cumulative distribution function for the NPV of the project is $F(NPV)$, the NPV at risk for a confidence level α and the confidence level for $NPV = 0$ can both be found using percentile analysis (Figure 3).

When the decision criterion is not found to be normally distributed, Monte Carlo simulation can be employed to generate a possible distribution for the criterion. Multiple NPV scenarios for the project can be generated with this technique, using different samples for the input variables (from historical data or from a user-defined distribution). The generation process can be repeated as many times as desired. The obtained results ($NPV_1, NPV_2, \dots, NPV_n$) can be sorted in ascending order and thus the cumulative distribution function can be found. The distribution function can be estimated from the empirical distribution:

$$F_n(NPV) = \frac{(\#NPV_i \leq NPV)}{n} \quad (5)$$

Eq. (5) corresponds to the relative frequency of the NPV , where $\#NPV$ is the number of results of the NPV_i obtained from the simulation that are lower than a specified NPV value. Therefore, the NPV at risk for a certain confidence level can be calculated using the corresponding percentile:

$$F_n^{-1}(\alpha) = NPV_\alpha \quad (6)$$

The confidence level for $NPV = 0$ can be found with the probability that NPV is lower than or equal to zero [$P(NPV \leq 0)$], which would be:

$$F_n(0) = \frac{(\#NPV \leq 0)}{n} \quad (7)$$

2.8 Definition of decision variables and associated constraints

The next step in the optimization process is the definition of the decision variables associated to the model. In portfolio selection problems, the decision variables can be of two kinds. If we talk about investments such as stocks or bonds for example, the key decision variable is the percentage of budget allocation to each type of investment. However, for portfolios of real projects such as those presented here, the decision variables are defined as binary. The variable takes the value 0 if the project is not selected, and it takes the value of 1 if the project is part of all projects.

Only one type of budget constraint is considered, since it has a budget allocated by the company, which is generally not sufficient to meet the needs of total investment.

2.9 Outputs of the optimization model

The aim of the model is to find the optimal set of projects in a way that the ENPV of the portfolio is optimized, complying with all the financial constraints. The usual practice in these financial optimization processes is the specification of requirements for the projected variables from goals established by the organization itself. For example, the maximum *NPV at risk* allowed for a given confidence level, should be specified.

3. Results and discussion

An applied case was developed to illustrate the proposed methodology. The information came from a public utility company. As part of their management processes, utilities companies must make up banks of projects in which the information related to the assessment and management of individual projects is condensed to preserve the memory of each project and in this way, the management of the projects can be optimized.

A subset of ten projects from the 2007 investment portfolio of the company was selected. The selected projects were defined in terms of the following parameters: Revenue growth rate,

earnings before interest, taxes, depreciation and amortization (EBITDA) margin, initial investment and working capital increase. The model considered the cash flow after paying all the applicable taxes.

The utility company that served as an example for this paper has a Technical Planning Department, comprised by about 20 engineers. They are dedicated to the configuration and selection of projects for the expansion and optimization of their networks. These engineers meet and try to estimate (by consensus building) the values of the variables required for the evaluation of the projects. Of the values defined by the group of engineers, one can obtain the minimum value, the more likely and the maximum for each of the key variables of the model. Afterwards, the values obtained are used as the vertices of a triangular distribution for the model that finally enter the data.

The triangular distribution is used because it is intuitively easy for the analyst to come up with the three required values: The minimum and maximum (which can be identified with best-and-worst case scenarios) and the most likely value (the analyst's best guess). The benefits of using this type of distribution have been widely discussed in the literature on financial risk. (Johnson (1997), Williams (1992)).

Table 1. Model Entry Variables

Project	Revenue Growth Rate			EBITDA Margin			Working Capital			Decision Variable
	Min %	Lik %	Max %	Min %	Lik %	Max %	Min %	Lik %	Max %	
1	8	10	11	40	42	45	6	10	12	(0,1)
2	9	10	15	38	40	42	9	12	13	(0,1)
3	8	9	13	47	48	53	13	15	19	(0,1)
4	10	12	14	45	50	52	15	17	18	(0,1)
5	17	18	19	37	40	43	9	10	11	(0,1)
6	10	11	12	31	32	33	7	8	10	(0,1)
7	8	9	13	44	46	47	4	5	6	(0,1)
8	10	13	15	43	47	49	4	7	9	(0,1)
9	8	10	12	41	43	45	3	6	7	(0,1)
10	4	6	8	33	35	36	8	10	14	(0,1)
Tax Rate	30%									
Depreciation period (years)	4									
Available budget	15000									
Cost of capital	14%									

Table 2. Cash Flow Model Projects

Cash Flow Project i	Year 0	Year 1	Year 2	Year 3	Year 4
Revenue		Rev 1	Rev 2	Rev 3	Rev 4
EBITDA		EBITDA 1	EBITDA 2	EBITDA 3	EBITDA 4
Depreciation		Dep 1	Dep 2	Dep 3	Dep 4
EBIT		EBIT 1	EBIT 2	EBIT 3	EBIT 4
Taxes		Taxes 1	Taxes 2	Taxes 3	Taxes 4
NOPAT		NOPAT 1	NOPAT 2	NOPAT 3	NOPAT 4
+Depreciation		Dep 1	Dep 2	Dep 3	Dep 4
CAPEX	Capex				
WC Increase		WC_Increase 1	WC_Increase 2	WC_Increase 3	WC_Increase 4
Cash Flow Available	Cash Flow 0	Cash Flow 1	Cash Flow 2	Cash Flow 3	Cash Flow 4

Table 3. Main Economic Indicators for each Project

Project	NPV Inflows (Cost of capital)	NPV (Cost of capital)
1	1989.9	159.8
2	1815.7	-74.3
3	2309.1	-70.9
4	2035.6	-104.4
5	2232.5	-137.5
6	1704.0	114.0
7	2705.8	205.8
8	2282.8	232.8
9	1952.2	-27.8
10	1845.4	145.4
E(NPV) Portfolio		443

Table 1 presents the parameters and variables for the model. In addition to the elements specified before, the depreciation period for each project, the available budget, the income tax rate, and the cost of capital are inputs used for the analysis of the projects.

Once the input variables and parameters of the project evaluation model were defined, the next step was the development of the after-tax cash flow for each project during the analysis period. After this, the NPV was calculated for each project in the base scenario. Table 2 presents the cash flow structure applied for each project.

Then the ENPV for the portfolio was calculated (Table 3). The available budget was only COP\$15000 million, and selecting all projects would require a total initial investment of COP\$20420 million. Thus, the problem was to determine which projects to select to maximize the total ENPV while staying within the budget limitation, considering uncertainty conditions associated with the main entry variables of the model.

Also, a factor of budgetary efficiency was incorporated into the model, which is typical of public companies, as the allocation of resources requires a minimal implementation of the approved budget. In this case, the minimum factor in the analysis of budget execution was 80% (Eq. 8). Similarly, the model is obviously included in the budget restriction.

Table 4. Output Variable Net Present Value (NPV)

	Project								
	1	2	3	4	5	6	7	8	9
Mean	118.4	-15.9	-50.2	163.2	5.8	72.3	96.5	143.0	1.0
Median	112.8	-18.3	-55.9	169.8	8.2	71.9	93.1	139.3	-3.0
Standard Deviation	95.5	87.1	136.6	125.5	117.9	50.7	69.8	104.2	71.8
Variance	9115.6	7594.4	18671.5	15761.9	13909.3	2566.0	4868.5	10858.5	5194.4
Skewness	0.2	0.1	0.1	-0.3	-0.1	0.0	0.1	-0.1	0.2
Kurtosis	2.9	2.8	2.6	2.6	2.4	2.6	2.8	2.7	2.8
Coeff. of Variability	0.8	-5.5	-2.7	0.8	20.2	0.7	0.7	0.7	73.7
Minimum	-156.5	-257.8	-423.7	-209.6	-275.3	-59.9	-110.3	-150.1	-200.4
Maximum	433.9	233.6	307.0	446.8	321.7	202.9	296.2	428.4	211.8
Mean Std. Error	3.0	2.8	4.3	4.0	3.7	1.6	2.2	3.3	2.3
Prob (NPV>0)	89.8%	43.6%	34.9%	88.2%	51.9%	91.8%	91.7%	91.1%	48.1%
Prob (NPV<0)	10.2%	56.4%	65.1%	11.9%	48.1%	8.2%	8.3%	8.9%	51.9%

The sum of the initial investment of the approved projects cannot exceed the amount of available resources

$$80\% \times Budget \leq \sum_{i \in S} I_i \leq Budget \quad (8)$$

To analyze the results obtained in addition to the total portfolio, each project was evaluated individually, with the purpose of studying the risk profiles of each project, and for analyzing its impact on the recommended portfolio, according to each of the studied objective functions. Table 4 shows the simulation results for each project under evaluation. Note that the projects present different profiles in terms of key outcomes such as ENPV and the level of risk measured by the probability of obtaining negative values.

For example, if project 3 is compared with project 5, we notice that the level of risk taken in project 5 is much larger when considering issues such as the coefficient of variation that relates the mean and standard deviation of the NPV of the project. But when assessing the possibility of negative results in the NPV, project 5 has a probability of 48.1% compared to 65.1% of project 3.

Figure 4 shows the results obtained from the simulation of the total NPV of the project portfolio, without assuming any kind of budget constraints. In this scenario, which exceeds the available budget, the risk of non-feasibility of the portfolio was estimated at 3.55%.

By continuing with the proposed methodology, the next step was to review the composition of portfolios, improved from defined objective functions. The goals pursued with this process were geared towards improving the overall risk and portfolio ENPV. Figures 5, 6 and 7 show the results obtained with different portfolios from each of the objective functions.

Different optimization routines were tried using OptQuest in CrystalBall to search for optimal values for the budget allocation decision variables. Three different objective functions were evaluated for comparison purposes. These results are presented in Table 5.

It can be noticed that the objective functions that maximize the ENPV of the portfolio suggest that the budget allocation should be close for all the projects. However, when a risk function such as the coefficient of variation (measured as the relationship between the mean and the standard deviation of the results) is considered, the results change considerably. Companies usually define the maximization of value as their objective function; however, it is necessary to consider that this objective can be accomplished through different levels of risk. In consequence, the decision makers should consider the risk element through some relevant measure. In this example, this consideration was achieved maximizing the 2th percentile of the weighed NPV distribution.

Table 5 shows that when maximizing the average NPV, the efficiency factor of the budget is lower, close to 80%, while the option of maximizing the probability that the portfolio ENPV is greater than zero, improves the efficiency indicator budget to 94.8%. What is interesting in the latter recommended portfolio, in addition to the improvement of the efficiency factor of the budget, is the reduction of the risk of non-feasibility of the project portfolio. Additionally, the expected value of the portfolio under this objective function is reduced to only COP\$ 608 million, i.e. 3.5%. The recommendation to improve the risk profile of the portfolio looks reasonable to meet the proposed objectives without significantly sacrificing the average value of the portfolio.

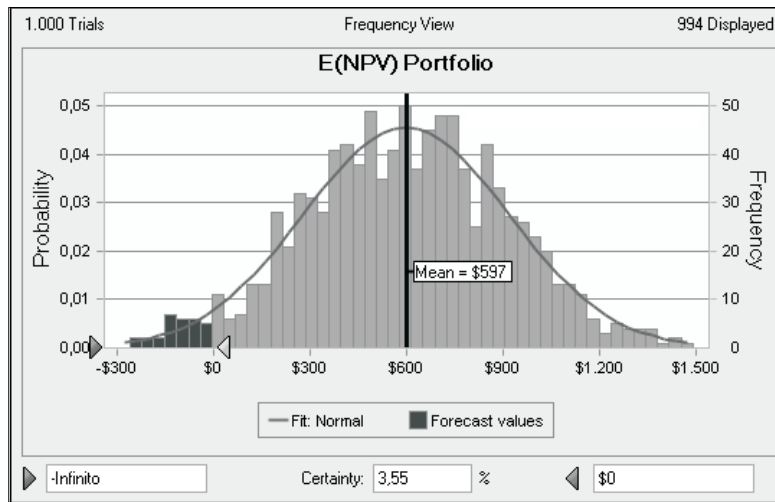


Figure 4. Simulation Results (1000 trials)

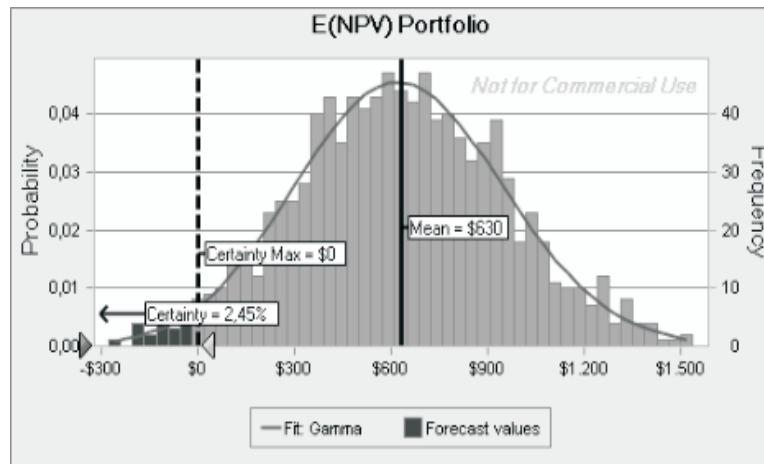


Figure 5. Simulation Result – Portfolio with Objective Function Max E(NPV)

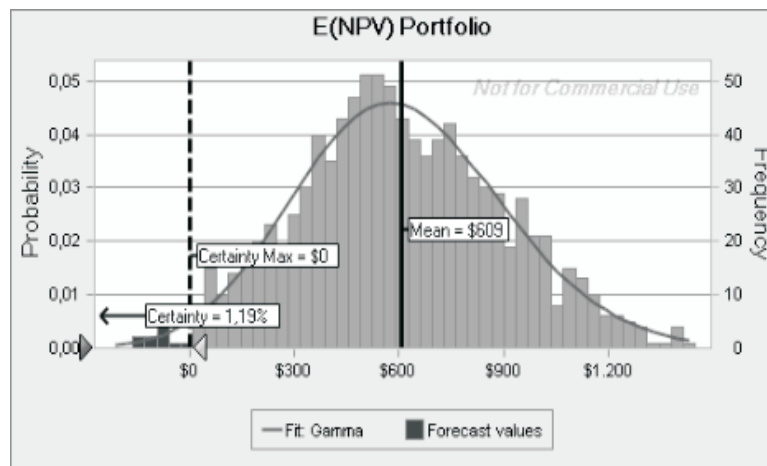


Figure 6. Simulation Result – Portfolio with Objective Function Max Percentile 2 NPV Portfolio

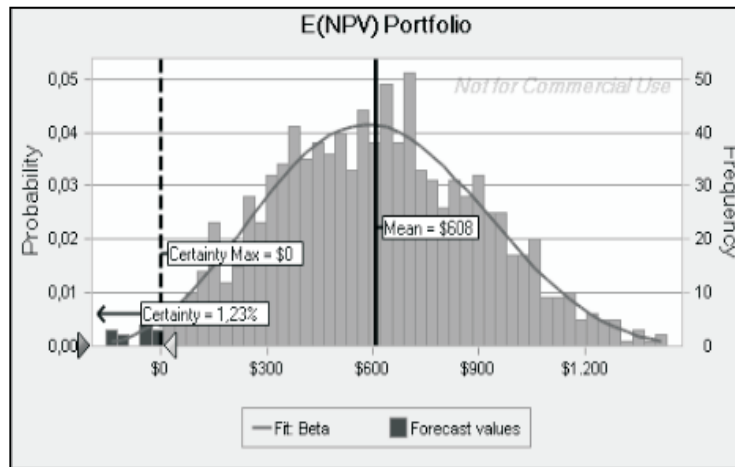


Figure 7. Simulation Result – Portfolio with Objective Function Max Prob ($NPV \geq 0$)

Table 5. Comparison of three objective functions

Portfolio	Project										Objective	Budget % Efficiency
	1	2	3	4	5	6	7	8	9	10		
1: Max mean NPV portfolio	1	1	0	1	0	0	1	0	1	1	630**	80.3
2: Maximize the 2% Percentile of E(NPV) Portfolio	1	0	0	0	1	0	1	1	1	1	21**	82.9
3: Maximize the Probability that E(NPV) Portfolio is greater than \$0	1	1	1	0	0	1	1	1	1	0	98.3%	94.8

** Millions \$

4. Conclusions

This paper contributes to the existing literature on performance evaluation in capital budgeting settings by demonstrating the crucial role of properly designing income recognition rules in the presence of capital constraints. In situations with a limited budget or alternative projects, it is desired to select the NPV-maximizing investment portfolio.

The consideration of quantitative risk analysis in portfolio management opens the possibility to set standards in the evaluation of different kinds of projects. The definition of risk in the portfolio optimization analysis can significantly affect the portfolio selection. In this way, the scenario analysis of the maximum financial exposure gives a sense of what could happen to the value of the company by changing the risk tolerance for new ventures.

For further analysis, models with a higher degree of complexity should be considered. In addition to economic uncertainty and budgetary constraints, more

constraints of technical and structural nature could be included, such as relations of precedence in the scheduling of certain projects, staffing requirements, energy consumption, and possible synergistic relationships between projects to be undertaken, among other alternatives. The use of stochastic optimization techniques, as suggested in this paper, may well be applicable in other decisions which are considered necessary to include elements that generate uncertainty.

While the definition of the optimal set of projects that a company should develop usually depends on the economic factors associated with the generation of value, other qualitative criteria that attempt to weigh other factors associated with the development of the project are often used. For instance, the social impact and the improvement of product quality are among these factors. In these cases it is desirable to use other tools and techniques like the Analytical Hierarchy Process (AHP) that take into account these aspects. Although our example focuses primarily on maximizing the expected NPV, there is evidence that managers in industry consider alternate measures such as IRR, payback

period, and return duration, along with NPV, when making capital budgeting decisions (Barney & Danielson, 2004). There is also evidence that non-financial criteria can also play an important role in the ultimate decision to invest in a project.

6. References

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